The Geodesy Corner

WHAT IS HEIGHT ANYWAY?

BY JAMES FERGUSON

I recently had a call from a colleague concerning a previous Geodesy Corner item entitled "Datums and Ellipsoids - Are They The Same?". My friend noted a few areas where the concepts were unclear, and suggested clarifying these points in a future Geodesy Corner.

It was encouraging for me to hear from this reader for two reasons - to know that people are reading the articles, and for the more important reason that a fellow professional surveyor was attempting to uphold a standard in another surveyor's work. Alors, before we get to the subject of heights, some explanations.

In the article in question, I was outlining some of the basic relationships between datums and ellipsoids, and began with the definition of a datum. I was trying to be quite general when I stated that a horizontal datum had an origin and orientation, and was extended through many types of survey methods - ie. new points are referenced to the datum by attachment to old points. With the introduction of the new NAD83 datum (vs NAD27), we can say that we now have two definitions of a datum - the classical NAD27 approach of an origin and orientation, and the more modern NAD83 approach where the entire system has an origin, and cartesian coordinate axes (X,Y,Z). Each of these axes have a specific orientation, for instance the spin axis of the earth.

In addition, for the latter modern approach all coordinates are referenced to what may be called a "coordinate surface", and new points are referenced to the this "system oriented" surface. In a nutshell, be aware of the existence of the two types of datum definitions.

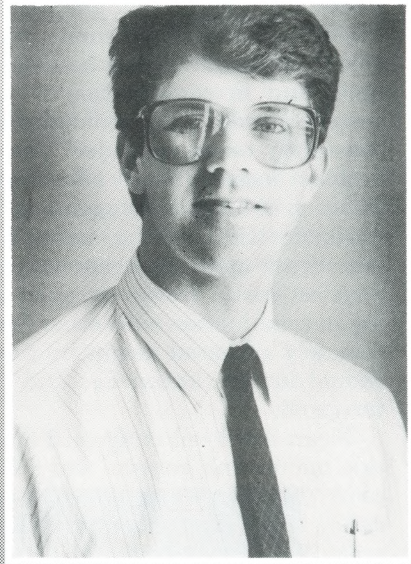
A second point of clarification involves the descriptions of the radius of curvature in the prime vertical, and the radius of curvature in the meridian. These two components combine to form the angular difference known as the deflection of the vertical, and as such are orthogonal to each other.

Therefore, we can say that the radius of curvature of the meridian is the projection of the deflection of the vertical onto the meridian plane, while the radius of curvature in the prime vertical is the projection of the deflection angle onto the prime vertical. The quantities vary in accordance with the change between the geoid and the reference ellipsoid.

The final clarification is to say that UTM coordinates are not based specifically on the NAD27 datum, although until the advent of NAD83 UTM coordinates in North America were referenced to the NAD27 datum. Essentially, the UTM system is global in nature and can be based upon any reference ellipsoid thus specified.

Now, "What is Height Anyway?".

Let me first describe several elements which are important when dealing with heights. As I have discussed, a horizontal datum is a reference to which horizontal survey observations can be related to obtain coordinate information for new points. Similarly, a vertical datum is a reference to which vertical observations can be related to obtain heights, or elevations. Associated with this vertical datum is the geoid, defined as the best representation of mean sea level (MSL) over the earth. The geoid is defined by MSL - which is calculated from global height information - as a first approximation, sufficient for our purposes.



HEIGHT cont'd

One of the major problems associated with heights is the computation of the geoid. It has only been recently that acceptably accurate models have been possible.

Using methods such as Stokes Integral and Spherical Harmonic coefficient models (what a mouthful), gravity information of the earth is used to model the geoid. With the advent of satellite technology, more reliable gravity information has become available, and in combination with terrestrial gravity data, geoid models are constantly being upgraded. The geoid problem doesn't directly affect the computation of heights using say, spirit or trigonometric levelling, because they are direct methods observed on the physical earth. However, spirit levelling in combination with other forms of remotely gathered height information should involve knowledge and use of the geoid.

Mean sea level is arrived at using tide monitoring gauges, and different combinations of gauges may be used to define MSL for a particular geographic area. There are several gauges along Canada's ocean coasts which are continually monitoring the ocean activity. It has been estimated that the actual difference in MSL from the east to west coast is on the order of 20 centimetres. Data from the tide gauges is used after observations have been taken over long periods of time. In doing so, long term oscillations of the sea level can be determined and accounted for. The transfer of the tide gauge data to shore based reference marks results in the establishment of benchmarks.

Mean sea level height, or orthometric height is the most common form of height. Other types of height include ellipsoidal heights, normal orthometric heights and dynamic heights, among others.

The latter two forms of height are really variations on the orthometric height, thus our discussion will rest with orthometric and ellipsoidal heights. Before we delve into these two, some insight into vertical datums in Canada.

Serious levelling work in Canada began in the late 1800's, and a major compilation of levelling data was done in the 1920's. This compilation provided a first national vertical datum known as the North America Vertical Datum of 1929, and used MSL heights obtained from tide gauges on the east and west coasts of Canada. The 1929 datum is currently being used today, as we await the re-compilation of vertical data to be known as the North American Vertical Datum of 1988 (NAVD88). Other interim datums have been computed and used throughout North America, such as the International Great Lakes of 1955 which established a standard for use by the United States and Canada along the St. Lawrence seaway and the great lakes. In many instances local datums are used within a predefined area, and may be based on a series of benchmarks thought to be of good quality. I have seen cases where two adjacent municipalities use local vertical datums which differ by 10 or 20 centimetres. Caveat Emptor!

To a surveyor, orthometric height is the most useful form of vertical information, and is the height obtained when spirit levelling is done. In actual fact, spirit levelling yields height differences which translate into heights when applied to known vertical stations, or benchmarks. Trigonometric levelling also yields height differences, and "trig" levels are obtained using measured zenith angles to a target and knowledge of the deflection of the vertical (Laplace correction) at each point. Trigonometric levelling is much less accurate than spirit levelling, and is very sensitive to the affects of atmospheric refraction.

As with horizontal surveys, vertical surveys must meet certain standards to be classified as a certain order of survey. Accuracies range from the sub millimetre level for "special order" surveys to the centimetre or greater level for fourth order surveys. The Department of Energy Mines and Resources is one agency which issues standards for vertical surveys. These are normally described as accuracies required in terms of the distance over which the levels were run.

Ellipsoidal height is a height measurement referenced to a particular ellipsoid, say the GPS reference ellipsoid WGS84. Since an ellipsoid is a mathematical model, we cannot obtain ellipsoidal heights in the same direct manner that we can obtain orthometric heights, ie. through spirit levelling. Satellite observations can however, give us heights that are ellipsoidal. This is because satellite systems are referenced to a specific ellipsoid, and all resultant coordinate information is with respect to the ellipsoid. The question then arises of how to relate ellipsoidal heights with spirit levelled heights. This problem boils down to the relationship between the ellipsoid in question, the geoid, and the orthometric height.

The orthometric height is the height of a point above the geoid. The ellipsoidal height is the height of the point above the ellipsoid. Since the geoid and the ellipsoid are not generally coincident (they are coincident in certain geographic areas) there is a separation between the geoid and the ellipsoid. This separation is known as the geoidal height, or undulation of the geoid. We will use the term undulation from here on. Depending on the reference ellipsoid being used for computations or observation gathering, the value of the undulation can vary by an order of magnitude. For example, a point located in the middle of Ontario may have an undulation value of eight (8) metres when referenced to the Clarke 1866 ellipsoid, but when referenced to the GRS80 ellipsoid (used for NAD83) the undulation may be on the thirty-five (35) metre level, and with opposite sign. The sign of the undulation value tells whether or not the geoid passes above or below the reference ellipsoid at any particular point. If the value of the undulation is zero (0), this indicates that the geoid is in fact coincident with the reference ellipsoid in question. Note that for most applications, the difference in undulation between sur-

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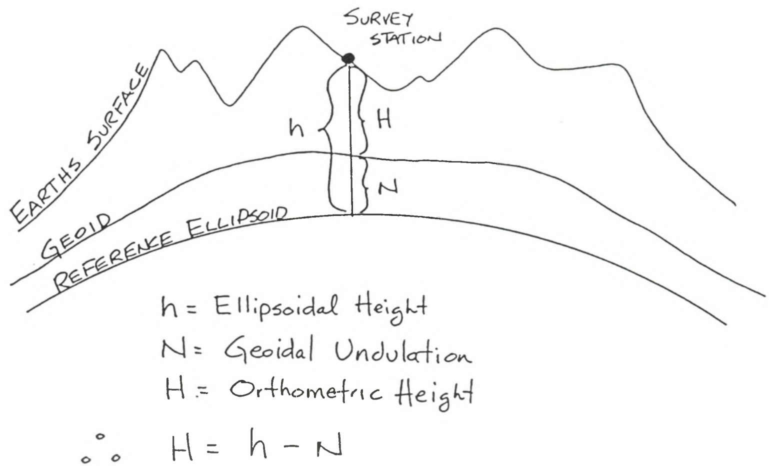
vey points is more important than the absolute value of the undulation at a point. The absolute accuracies of present day geoid models are on the one metre level.

With the emergence of the integration of satellite vertical data and conventional levelling data, the application of the geoidal undulation to a survey problem is becoming increasingly important. In a rigorous least squares adjustment process, where it is now possible to combine all types of survey information, care must be taken to address the problem of the relationships between ellipsoids, orthometric heights and the geoid. Using satellite systems such as GPS will produce heights referenced to the ellipsoid; conventional levelling gives us orthometric height; and the link between them is the geoidal undulation. Information regarding the latter can be found in interpolation tables published by Energy Mines and Resources, or in software published by several private companies.

The following diagram will help to illustrate the relationship between the various elements I have presented in this article.



If you have any suggestions, queries or comments regarding the Geodesy Corner, please write to me care of the Association offices.

Next time in the Geodesy Corner - "Control Surveys - Who needs them?"